# Chapter 24

1

## Gauss's Law



1

# Lecture 01

2

## Electric Flux



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# Electric Flux

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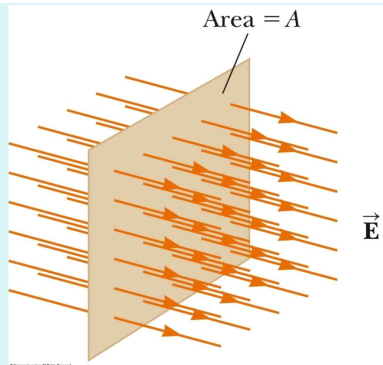
Electric flux is the product of the magnitude of the electric field and the surface area, $A$, perpendicular to the field.

$$\Phi = EA$$



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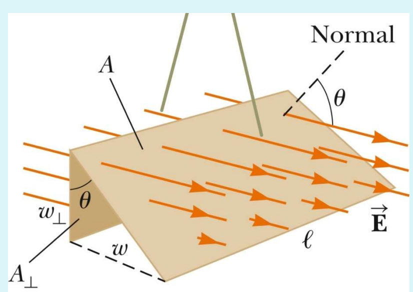
# Electric Flux, General Area

4

- The electric flux is proportional to the number of electric field lines penetrating some surface.
- The field lines may make some angle $\theta$ with the perpendicular to the surface.
- Then:

$$\Phi = EAcos\theta$$

$$\Phi = \vec{E} \cdot \vec{A}$$



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# Electric Flux, Interpreting the Equation

5

Units: $N \cdot m^2/C$

A scalar quantity

$\theta$: the angle between the electric field and the normal to the surface.

- The flux is a maximum when the surface is perpendicular to the field. ($\theta = 0°$)
- The flux is zero when the surface is parallel to the field. ($\theta = 90°$)



# Electric Flux, General

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- In the more general case, look at a small area element.

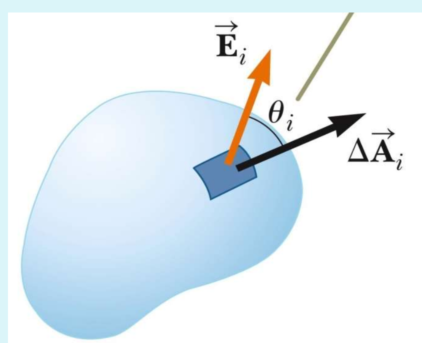
$$\Delta\Phi = E_i \Delta A_i cos\theta_i = \vec{E}_i \cdot \Delta\vec{A}_i$$

- In general, this becomes

$$\Phi = \lim_{\Delta A_i \to 0} \sum \vec{E}_i \cdot \Delta\vec{A}_i$$

$$\Phi = \oint \vec{E}_i \cdot d\vec{A}_i$$

- The surface integral means the integral must be evaluated over the surface in question.



# Electric Flux, Closed Surface

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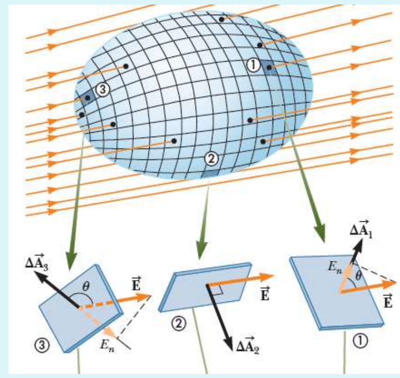
- Assume a closed surface
- The vectors $\Delta\vec{A}_i$ point in different directions.
    - At each point, they are perpendicular to the surface.
    - By convention, they point outward.



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# Flux Through Closed Surface, cont.

8

$$\Phi \text{ is } \begin{cases} (1)\ \textit{positive}\ ;\ 0^\circ \le \theta < 90^\circ\ ; \\ \text{the field lines are crossing the surface} \\ \text{from the inside to the outside} \\ \\ (2)\ \textit{zero}\ ;\ \theta = 90^\circ\ ; \\ \text{the field lines graze surface} \\ \\ (3)\ \textit{negative}\ ;\ 90^\circ < \theta \le 180^\circ\ ; \\ \text{the field lines are crossing the surface} \\ \text{from the outside to the inside} \end{cases}$$



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# Flux Through Closed Surface, final

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- The net flux through the surface is proportional to the net number of lines leaving the surface.
  - This net number of lines is the number of lines leaving the surface minus the number entering the surface.
- If **E** is perpendicular to the surface, then

$$\Phi = \oint \vec{E}_i \cdot d\vec{A}_i = \oint E \, dA$$

  - The integral is over a closed surface.

# Flux through a disk

Friday, 29 January, 2021 20:47

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

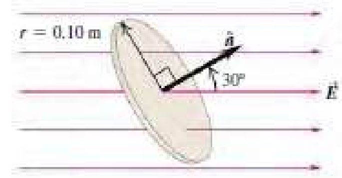

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.

H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.


A disk of radius $0.1\ m$ is oriented with its normal unit vector $\hat{n}$ at $30^{\circ}$ to a uniform electric field $\vec{E}$ of magnitude $2 \times 10^3\ N/C$.




- What is the electric flux through the disk?
- What is the flux through the disk if it is turned so that $\hat{n}$ is perpendicular to $\vec{E}$?
- What is the flux through the disk if $\hat{n}$ is parallel to $\vec{E}$?

Solution:

The area is

$$A = \pi r^2 = \pi(0 \cdot 1)^2 = 0.0314 m^2$$

and the angle between $\vec{E}$ and $\vec{A}$ is

$$\theta = 30^{\circ} \ ,$$

$$\Phi = EA\cos\theta = 2 \times 10^3 \times 0.0314 \times \cos 30^{\circ} = 54 N \cdot m^2/C$$

Solution:

The normal to the disk is now perpendicular to $\vec{E}$ , so $\theta = 90^{\circ}$ , $cos 90^{\circ} = 0$, and

$$\Phi = 0$$

Solution:

The normal to the disk is parallel to $\vec{E}$ , so $\theta = 0^{\circ}$ and $cos 0^{\circ} = 1$:

$$\Phi = EA\cos\theta = 2 \times 10^3 \times 0.0314 \times 1 = 63 N \cdot m^2/C$$

# Flux through a cube

Friday, 29 January, 2021 20:49

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.
R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
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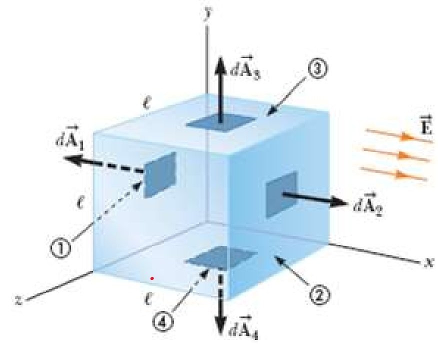
H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

Consider a uniform electric field **E** oriented in the positive x direction in empty space. A cube of edge length $\ell$ is placed in the field, oriented as shown. Find the net electric flux through the surface of the cube.




SOLUTION

Notice that the electric field lines pass through two faces perpendicularly and are parallel to four other faces of the cube. The flux through four of the faces (3,4, and the unnumbered faces) is zero because $\vec{E}$ is parallel to the four faces faces and therefore perpendicular to $d\vec{A}$ on these faces.

Write the integrals for the net flux through faces 1 and 2 :

$$\Phi_E = \int_1 \vec{E} \cdot d\vec{A} + \int_2 \vec{E} \cdot d\vec{A}$$

For face 1, $\vec{E}$ is constant and directed inward but $d\vec{A}_1$ is directed outward ($\theta = 180°$). The flux through this face:

$$\int_1 \vec{E} \cdot d\vec{A} = \int_1 E\cos 180° dA = -E\int_1 dA = -EA = -E\ell^2$$

For face 2, $\vec{E}$ is constant and outward and in the same direction as $d\vec{A}_2$ ($\theta = 0°$). The flux through this face:

$$\int_2 \vec{E} \cdot d\vec{A} = \int_2 E\cos 0° dA = E\int_2 dA = +EA = E\ell^2$$

Find the net flux by adding the flux over all six faces:

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = 0$$

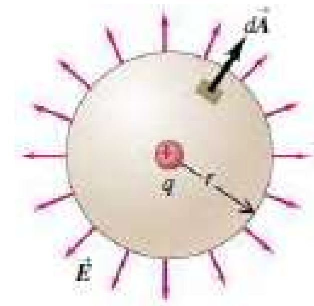
# Flux through a sphere

Friday, 29 January, 2021 20:48

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A point charge $q = +3\,\mu C$ is surrounded by an imaginary sphere of radius $r = 0.2\,m$ centered on the charge. Find the resulting electric flux through the sphere.




Solution:

The magnitude of the electric field is the same everywhere on the surface of the sphere $\left(E = \frac{1}{4\pi\varepsilon_\circ}\frac{q}{r^2}\right)$.

The field lines are perpendicular to the surface at every point on the surface $(\theta = 0°)$.

Because the Gaussian surface is spherical, then: $(A = 4\pi r^2)$. So:

$$\Phi = \oint \vec{E}\cdot d\vec{A} = \oint (E\cos\theta)dA$$

$$\Phi = E(\cos 0)\oint dA = E(\cos 0)A$$

$$\Phi = \frac{1}{4\pi\varepsilon_\circ}\frac{q_{in}}{r^2}(4\pi r^2) = \frac{q_{in}}{\varepsilon_\circ}$$

$$\Phi = \frac{3\times10^{-6}}{8.85\times10^{-12}} = 3.4\times10^{5}\ N.m^2/C$$